

A Data-driven Distributionally Robust Operational Model for Urban Integrated Energy Systems

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Abstract—A multi-energy conversion can effectively increase the utilization of renewable energy in the urban integrated energy system (UIES). Meanwhile, the uncertainties of renewable energy resources (e.g., wind energy) also bring increased challenges to the operation of UIES. In this study, a typical two-stage data-driven distributionally robust operation (DDRO) model based on finite scenarios is proposed for UIES including power, gas and heat networks to obtain a salient strategy from both an economic and robustness perspective. In the first stage, the forecasted information for wind power is especially included to improve the economic aspect of robust decisions. The worst probability distribution for the selected known real-time wind power scenarios can be identified in the second stage where the power differences caused by the real-time uncertainties of wind power can be mitigated by flexible regulation of energy purchasing and coupling units (such as gas turbine, power to gas equipment, electric boiler and gas boiler). Moreover, norm-1 and norm-inf co-constraints are utilized to construct a confidence set for the probability distributions of uncertain wind power. The whole two-stage model is solved by the column-and-constraint generation (CCG) algorithm. Finally, case studies are conducted to show the performance of the proposed model and various approaches.

Index Terms—Data-driven methods, distributionally robust optimization, urban integrated energy system, wind power.

NOMENCLATURE

A. Indices

i, j, k	Index of buses (nodes) for power (gas) system.
ij	Index of branches (pipelines) for power (gas) system.
t	Index of time period.
s	Index of scenarios.

B. Sets

$\delta(j)/\pi(j)$	Set of buses whose parent/child is bus j .
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Manuscript received December 14, 2019; revised March 19, 2020; accepted May 23, 2020. Date of online publication July 6, 2020; date of current version August 17, 2020. This work was supported by National Key Research and Development Program of China under Grant No. 2019YFE0111500, Science and Technology Department of Sichuan Province under Grant No. 2020YFH0040, and National Natural Science Foundation of China under Grant No. 51807125.

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DOI: 10.17775/CSEEJPES.2019.03240

Ω	Confidence set of Probability distribution for wind power probability distribution.
T	Set of time periods.
E	Set of branches.
E^g	Set of pipelines.
B	Set of power system buses.
N^g	Set of gas system nodes.
$N^{GT}/N^W/N^{GB}/N^{WHB}/N^{EB}/N^{ESS}$	Set of buses connected with GT/WTG/GB/WHB/EB/ESS.
N^{sub}	Set of buses connected with substation.
$N^{GS}/N^{GSS}/N^{P2G}$	Set of gas nodes connected with GS/GSS/P2G.
F_{11}	Operating total cost for GT.
F_{12}, F_{13}	Cost for wind power curtailed, energy purchase.
C_t^g/C_t^e	Price for gas/power purchase.
$y_{i,t}^{GT}/z_{i,t}^{GT}$	Cost of start-up/shut-down for GT.
$\alpha_i^{GT}/b_i^{GT}/c_i^{GT}$	Coefficient of power generation cost for GT.
δ^W	Penalty coefficient for wind power curtailed.
$c_t^{GT}/c_t^W/c_t^{sub}/c_t^{GS}$	Adjusted penalty coefficient for GT, wind power, power purchase and gas purchase.
r_{ij}/x_{ij}	Resistance/reactance of branch.
b_j	Shunt susceptance.
$P_{i,t}^{W,0}$	Forecast output for WTG.
$\alpha_c^{GSS}/\alpha_d^{GSS}$	Charge/discharge efficiency of GSS.
H_{GV}	High calorific value of natural gas.
$\eta_i^{GT}/\eta_i^{P2G}/\eta_i^{EB}/\eta_i^{GB}$	Conversion efficiency of GT/P2G/EB/GB.
λ_i^{GT}	Thermoelectric ratio between WHB and GT.
R_D^{GT}/R_U^{GT}	Ramp-down/ Ramp-up rate limit for GT.
$P_{j,t}^L/G_{i,t}^L/H_t^L$	Active power load, gas load and heat load.
$P_{i,t}^{GT}/P_{i,t}^W$	Day-ahead actual output for GT, WTG.
$u_{i,t}^{GT}$	Binary decision variable to indicate the on/off status of GT.
$y_{i,t}^{GT}$	Binary decision variable to indicate if GT turns on.

$z_{i,t}^{GT}$	Binary decision variable to indicate if GT shuts down.
P_t^{sub}/G_t^{GS}	Day-ahead power/gas purchase.
$P_{j,t}^{ESS,c}/P_{j,t}^{ESS,d}$	Charge/discharge power for ESS.
$P_{j,t}^{P2G}/P_{j,t}^{EB}$	Day-ahead power consumption for P2G, EB.
$Q_{j,t}^{GT}/Q_{j,t}^{sub}$	Day-ahead reactive power output for GT, substation.
$I_{ij,t}/V_{i,t}$	Current of branch and bus voltage.
$u_{i,t}^{ESS,c}/u_{i,t}^{ESS,d}$	Charge/discharge status for ESS
$(u_{i,t}^{GSS,c}/u_{i,t}^{GSS,d})$	(GSS).
$G_{i,t}^{GSS,c}/G_{i,t}^{GSS,d}$	Charge/discharge gas for GSS.
$G_{i,t}^{P2G}$	Day-ahead gas output for P2G.
$G_{i,t}^{GB}/G_{i,t}^{GT}$	Day-ahead gas consumption for GB /GT
$G_{ij,t}^{pipe}$	Gas flow for pipeline.
$\pi_{i,t}$	Pressure of gas.
$H_{i,t}^{GB}/H_{i,t}^{WHB}/H_{i,t}^{EB}$	Day-ahead heat output for GB /WHB /EB.
$P_{i,t,s}^W$	Wind power output.
$\Delta(\bullet)$	Second-stage adjustable variables
$\bullet_{max}/\bullet_{min}$	Upper/lower bound of a quantity.

I. INTRODUCTION

WITH the decrease of fossil energy and the public awareness of environment protection, wind power, as one of the sustainable energy resources, has attracted an increasing amount of attention in many urban regions around the world. But wind power surplus and its uncertain nature have brought significant challenges to urban power distribution system operations, where a large amount of wind power must be curtailed to maintain the electric power supply-demand balance [1]. An urban integrated energy system (UIES) has been widely regarded as a promising way to improve the efficiency for wind energy utilization owing to the complementary and coupling characteristics of multi-energy networks including electricity, heat, and natural gas [2], [3].

In the past decade, a large number of studies on the co-optimization operation for integrated energy systems have been carried out [4]–[7]. Pan *et al.* [4] formulates an integrated quasi-dynamic model of electricity and heating systems considering security. A two-stage optimization method for a coupled capacity planning and operational problem for a regional integrated energy system is proposed in [5]. In [6], a low-carbon economic dispatch model is set up considering the information privacy of the electricity system and heating system. Dong *et al.* [7] proposed a multi-objective day-ahead optimal scheduling model considering the power needed to the gas operating costs. However, the uncertainty of renewable energy in UIES was not taken into account in the above studies. The deterministic optimal operational models make decisions according to only the forecasted scenario, which cannot guarantee the security and economic optimization of the system in practical applications. In response to this problem, several methods have been suggested recently to address various uncertainties. Stochastic (SO) and robust optimization (RO) approaches are two common methods among these studies. SO

usually constructs its scenarios according to the probability distribution of uncertain parameters and the objective is to minimize the weighted average of costs for all scenarios [8], [9]. The scenario method is used to solve the economic dispatch problem with wind power in [8]. Wei [9] proposed a chance-constrained coordinated optimization approach for UIES considering the uncertainties in electricity loads, heat loads, and photovoltaic outputs. Compared with SO, RO does not need the probability distribution of uncertain parameters in advance and describes the uncertainty of parameters through an uncertain set. As long as the values of the parameters are within the range of the uncertain set, the solution of the robust optimization model is feasible. Since the worst-case scenario is considered, RO can improve the system robustness. Currently, RO approaches have been used to solve unit commitment [10] and power dispatch problems [11].

In practice, the probability distribution of wind power is complex and is impractical to be estimated accurately, which correspondingly affects the effectiveness of the SO approach. In addition, although the RO approach can guarantee the system robustness, it is usually too conservative because it minimizes the objective function in “the worst case” scenario [12]. Compared with SO and RO, the distributionally robust optimization (DRO) model not only requires no hypothetical probability distribution of uncertain parameters, but also reduces the conservativeness caused by extreme scenarios with low probability. It has been widely applied to unit commitment [13], [14] and reserve scheduling [15]. In addition to the traditional DRO method based on moment information, there are some other types of DRO methods based on KL-divergence [16] and Wasserstein distance [17]. But the duality principle and affine rules must be applied in the solving process of the above basic DRO model [16], [17], which leads to a higher complexity in the calculation process [18]. Motivated by this, we introduce the data-driven DRO (DDRO) approach to generate an optimization scheme for UIES in this study. The DDRO approach constructs the uncertain probability distribution confidence set based on wind power historical data without moment information. Moreover, the solution process of this method is much simpler. At present, the data-driven approach has been applied to unit commitment [19] and reactive power optimization [20]. Furthermore, Zhao [19] and Ding [20] only use norm-1 or norm-inf independently as constraints to construct the ambiguity set. This would lead to unilateral decisions and the dispatcher cannot decide which solution in norm-1 or norm-inf is better and more adaptable. In addition, much of the existing two-stage optimization approach research focuses on optimizing first stage unit commitment cost and second stage worst-case expected dispatch cost [21]. Obviously, only the unit commitment result for UIES is rather limited. The UIES operators must know the explicit day-ahead dispatch so that some regulation measures have to be taken on this basis to ensure the real-time supply and demand balance.

Based on the aforementioned discussions in this paper, we utilize norm-1 and norm-inf co-constraints to construct the confidence set for the uncertain probability distributions of some selected wind power scenarios. On this basis, a two-stage DDRO model for UIES is proposed to obtain a superior

We organize the remainder of this paper as follows. In Section II, we describe the characteristics of UIES and give its typical structure. The confidence set for the probability distributions of the uncertain wind power and two-stage DDRO model are constructed in Section III. In Section IV, the CCG algorithm, model linearization methods and the second-order cone relaxation (SOCR) method are described in detail. In Section V, case studies on a modified IEEE 33-bus system with a 6-node gas system are carried out. Finally, Section VI provides the conclusions of this study.

II. GENERAL STRUCTURE FOR UIES

UIES is one of main directions of future urban development. UIES refers to the integration of multiple network forms such as electricity, gas, and heat networks based on technologies such as power to gas (P2G) and combined heat and power (CHP) in the urban area where the multi-energy load demand is concentrated. UIES realizes a high penetration renewable energy utilization and improves the integrated energy efficiency based on coupling and complementary features among the different energy networks. Inefficient operation of equipment and resource waste would appear if multiple energy resources cannot be effectively scheduled. Thus, an optimal

operational scheme should be devised to improve the system's economic performance and efficiency.

A typical structure of the UIES is demonstrated in Fig. 1, which contains five parts: 1) Energy supply: main grid, the natural gas source (GS), gas turbine (GT) unit, the wind turbine generator (WTG); 2) Energy transmission: electricity network, gas network and heat supply pipelines; 3) Energy conversion: the power to gas (P2G) equipment, the waste heat boiler (WHB), the electric boiler (EB), the gas boiler (GB); 4) Energy storage: the electrical storage system (ESS) and the gas storage system (GSS); and 5) Energy consumption: Electrical, gas and heat loads.

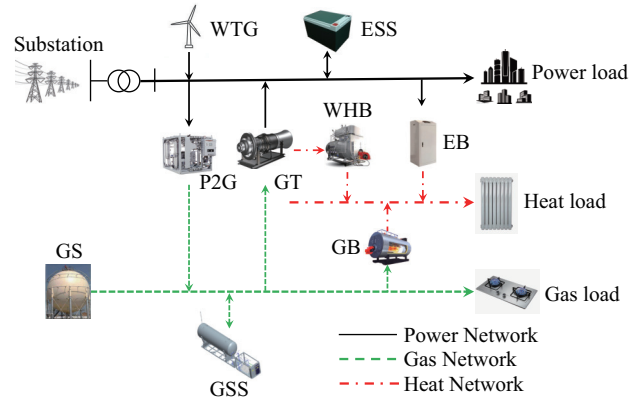


Fig. 1. The typical structure of UIES.

In Fig. 1, P2G technologies enable the energy flow direction from electricity to natural gas after two steps including hydrogen generation by electrolysis of water and reaction with CO₂. The GT and WHB form a CHP system, and the high-temperature flue gas generated in the GT power generation process is recycled and utilized by WHB to generate hot water, which can provide a heat supply. EB and GB convert the power and natural gas respectively into heat to satisfy the end-users' heat demand. In addition, there are various energy storage devices in UIES (ESS and GSS considered in this study). In this multi-energy system, wind power is preferentially used to meet the end-user energy demand because of its low-carbon feature. The surplus wind power can be stored or converted into other forms of energy through the above storage and coupling devices. Although wind power is renewable, the negative influence on the system operations should not be ignored due to its volatility and randomness. Therefore, we focus on the optimal operation of an urban integrated energy system considering the uncertainties of wind power.

III. TWO-STAGE DATA-DRIVEN DISTRIBUTIONALLY ROBUST OPERATION MODEL FOR UIES

In this section, the two-stage DDRO model is described in detail after the probability distribution confidence set of wind power scenarios is constructed.

A. Confidence Set

Due to its intermittent nature, it is difficult to precisely forecast wind power generation. The practical probability

distribution of wind power is unknown and difficult to obtain. Luckily, a large number of historical samples are available in the energy management system. Therefore, in this study we use a data-driven distributionally robust optimization method to model the uncertainties of wind power by constructing a confidence set for the ambiguous probability distribution of wind power based on the number of historical data. The detailed construction process is described as follows.

Assuming that there are M samples in a given set of wind power historical data, we can construct S typical scenarios by a clustering method. The number of each cluster is $M_1, M_2, \dots, M_S$ and $M = \sum_{s=1}^S M_s$. The base probability of each scenario can be obtained according to its proportion in the total samples, that is, $p_s^0 = M_s/M$, $s = 1, 2, \dots, S$. Because of the limited information from the historical data, the actual probability distribution cannot be exactly determined by this initial sample. Therefore, we allow the probability distribution of wind power to be arbitrary within a pre-defined confidence set Ω constructed from the historical data. The confidence set Ω is set up by using the norm-1 and norm-inf co-constraints to make the probability distribution fluctuation range more realistic, which is expressed as follows:

$$\Omega = \left\{ \{p_s\} \left| \begin{array}{l} p_s \geq 0, s = 1, \dots, S \\ \sum_{s=1}^S p_s = 1 \\ \sum_{s=1}^S |p_s - p_s^0| \leq \theta_1 \\ \max_{1 \leq s \leq S} |p_s - p_s^0| \leq \theta_\infty \end{array} \right. \right\} \quad (1)$$

where p_s represents the probability of scenario s and p_s^0 is its base probability according to historical data. θ_1 and θ_∞ denote the fluctuation limits (i.e., tolerance value) in norm-1 and norm-inf constraints, which can be set based on the following rule. According to [19], $\{p_s\}$ in the confidence set satisfies the confidence level.

$$\begin{cases} \Pr \left\{ \sum_{s=1}^S |p_s - p_s^0| \leq \theta_1 \right\} \geq 1 - 2S \exp(-2M\theta_1/S) \\ \Pr \left\{ \max_{1 \leq s \leq S} |p_s - p_s^0| \leq \theta_\infty \right\} \geq 1 - 2S \exp(-2M\theta_\infty) \end{cases} \quad (2)$$

The right-hand sides of the above inequality (2) are the probability distribution confidence level, represented as α_1 and α_∞ . Subsequently, θ_1 and θ_∞ are obtained as follows:

$$\begin{cases} \theta_1 = \frac{S}{2M} \ln \frac{2S}{1 - \alpha_1} \\ \theta_\infty = \frac{1}{2M} \ln \frac{2S}{1 - \alpha_\infty} \end{cases} \quad (3)$$

From (3), it can be seen that the θ_1 and θ_∞ are determined by the number of historical data M and the given confidence level. As the number of historical data M tends to infinity, the fluctuation limits θ_1 and θ_∞ decrease to 0.

B. Objective Function

The DDRO model for UIES in this study is divided into two stages. The power difference caused by the real-time uncertainties of wind power can be mitigated by flexible regulation measures (e.g., GT). By applying confidence set (1), the general two-stage DDRO model can be represented as follows:

$$\min F_1 + \max_{\{p_s\} \in \Omega} \left(\sum_{s=1}^S p_s (\min F_{2,s}) \right) \quad (4)$$

In the above model, F_1 denotes the first-stage cost, which is related to the day-ahead robust decision variables; $F_{2,s}$ is the second-stage adjustable cost caused by the uncertainties of wind power. The robust variables (without scenario index s) cannot be adjusted after the realization of the real-time uncertainties of wind power. In this study, the day-ahead robust variables include the electricity and the gas purchased from the upstream grid, the GT generation power and state, and the WTG generation. The electrical and gas storage systems are also included in the first stage decisions to obtain a better economic efficiency [22], [23]. It should be noted that the forecasted wind power is also included in the first-stage to help stipulate the robust operational scheme. The first-stage objective can be formulated as follows:

$$F_1 = F_{11} + F_{12} + F_{13} \quad (5a)$$

$$F_{11} = \sum_{t \in T} \left\{ \sum_{i \in N^{GT}} \{C_t^g [a_i^{GT} (P_{i,t}^{GT})^2 + b_i^{GT} P_{i,t}^{GT} + c_i^{GT} u_{i,t}^{GT}] + S_i^{GT, U} y_{i,t}^{GT} + S_i^{GT, D} z_{i,t}^{GT}\} \right\} \quad (5b)$$

$$F_{12} = \sum_{t \in T} \left(\sum_{i \in N^W} \delta^W (P_{i,t}^{W,0} - P_{i,t}^W) \right) \quad (5c)$$

$$F_{13} = \sum_{t \in T} (C_t^e P_t^{\text{sub}} + C_t^g G_t^{\text{GS}}) \quad (5d)$$

where F_{11} is the generation and unit commitment cost for GT; F_{12} is the wind power curtailment cost in the forecasted scenario; F_{13} represents the costs of electricity and gas purchasing from the power substation and GS in the forecasted scenario. The second-stage decision variables can be flexibly adjusted in the real-time operation where the worst-case scenario probability distribution for the given real-time wind power scenarios can also be identified. In this study, we only consider the adjustment of GT, power substation and GS. Therefore, the second-stage objective function can be formulated as follows:

$$F_{2,s} = \sum_{t \in T} \left[\sum_{i \in N^{GT}} c_{i,t}^{GT} \Delta P_{i,t,s}^{GT} + \sum_{i \in N^W} c_{i,t}^W (P_{i,t,s}^{W,0} - P_{i,t,s}^W) + c_{i,t}^{\text{GS}} \Delta G_{i,t,s}^{\text{GS}} + c_t^{\text{sub}} \Delta P_{t,s}^{\text{sub}} \right] \quad (6)$$

C. First-stage Constraints

In the first stage, the day-ahead scheme is scheduled under a forecasted wind power scenario. UIES should satisfy the power network constraints, gas network constraints, heating network constraints and power-gas-heat coupling constraints.

In the power network constraints, a branch flow model (BFM) is adopted as the power flow constraint.

1) Constraints of Power Network

a) *BFM*: The typical BFM is described in detail in [24]. After various coupling devices (e.g., GT, P2G, and EB) and storage systems are integrated, the power flow model can be expressed as follows:

$$\sum_{k \in \delta(j)} P_{jk,t} - \sum_{i \in \pi(j)} (P_{ij,t} - I_{ij,t}^2 r_{ij}) = (P_{j,t}^{\text{ESS},d} - P_{j,t}^{\text{ESS},c}) + P_{j,t}^{\text{GT}} + P_{j,t}^{\text{W}} + P_{j,t}^{\text{sub}} - P_{j,t}^{\text{P2G}} - P_{j,t}^{\text{EB}} - P_{j,t}^{\text{L}}, \forall t \in T, \forall j \in B \quad (7a)$$

$$\sum_{k \in \delta(j)} Q_{jk,t} - \sum_{i \in \pi(j)} (Q_{ij,t} - I_{ij,t}^2 x_{ij}) + b_j V_{i,t}^2 = Q_{j,t}^{\text{GT}} + Q_{j,t}^{\text{sub}} - Q_{j,t}^{\text{L}}, \forall t \in T, \forall j \in B \quad (7b)$$

$$V_{j,t}^2 = V_{i,t}^2 - 2(P_{ij,t} r_{ij} + Q_{ij,t} x_{ij}) + I_{ij,t}^2 (r_{ij}^2 + x_{ij}^2), \forall t \in T, \forall ij \in E \quad (7c)$$

$$I_{ij,t}^2 = \frac{P_{ij,t}^2 + Q_{ij,t}^2}{V_{i,t}^2}, \forall t \in T, \forall ij \in E \quad (7d)$$

b) *Wind Power*: Wind power output remains within the predicted output range.

$$0 \leq P_{i,t}^{\text{W}} \leq P_{i,t}^{\text{W},0}, \forall t \in T, \forall i \in N^{\text{W}} \quad (8)$$

c) *Substation Power*: Substation capacity limits the active power and reactive power.

$$\begin{cases} P_{i,t}^{\text{sub},\min} \leq P_{i,t}^{\text{sub}} \leq P_{i,t}^{\text{sub},\max} \\ Q_{i,t}^{\text{sub},\min} \leq Q_{i,t}^{\text{sub}} \leq Q_{i,t}^{\text{sub},\max} \end{cases}, \forall t \in T, \forall i \in N^{\text{sub}} \quad (9)$$

d) *Security Constrains*: Voltage and current should be within the appropriate range to ensure system security.

$$(V_j^{\min})^2 \leq V_{j,t}^2 \leq (V_j^{\max})^2, \forall t \in T, \forall j \in B \quad (10a)$$

$$0 \leq I_{j,t}^2 \leq (I_j^{\max})^2, \forall t \in T, \forall ij \in E \quad (10b)$$

e) *ESS*:

$$u_{i,t}^{\text{ESS},d} + u_{i,t}^{\text{ESS},c} \leq 1 \quad (11a)$$

$$0 \leq P_{i,t}^{\text{ESS},d} \leq u_{i,t}^{\text{ESS},d} P_i^{\text{ESS},\max} \quad (11b)$$

$$0 \leq P_{i,t}^{\text{ESS},c} \leq u_{i,t}^{\text{ESS},c} P_i^{\text{ESS},\max} \quad (11c)$$

$$E_{i,t+1}^{\text{ESS}} = E_{i,t}^{\text{ESS}} + \alpha_c^{\text{ESS}} P_{i,t}^{\text{ESS},c} - \alpha_d^{\text{ESS}} P_{i,t}^{\text{ESS},d} \quad (11d)$$

$$E_i^{\text{ESS},\min} \leq E_{i,t}^{\text{ESS}} \leq E_i^{\text{ESS},\max}, \forall t \in T, \forall i \in N^{\text{ESS}} \quad (11e)$$

where constraints (11a) indicate ESS cannot charge and discharge simultaneously; constraints (11b) and (11c) indicate the discharge/charge power limit for ESS; and constraints (11d) and (11e) are the capacity limitation constraints of the ESS.

2) Constraints of Gas Network

a) *Gas Balance*: The natural gas flow in the gas pipeline network needs to satisfy the law of conservation of mass.

$$G_{i,t}^{\text{GS}} + (G_{i,t}^{\text{GSS},d} - G_{i,t}^{\text{GSS},c}) + G_{i,t}^{\text{P2G}} = \sum_{j \in i} G_{ij,t}^{\text{pipe}} + G_{i,t}^{\text{GT}} + G_{i,t}^{\text{GB}} + G_{i,t}^{\text{L}}, \forall t \in T, \forall i, \forall j \in N^{\text{G}} \quad (12)$$

b) *Gas Injection Capacity*: The capacity of GS limits the gas purchased at time t .

$$G_{i,t}^{\text{GS},\min} \leq G_{i,t}^{\text{GS}} \leq G_{i,t}^{\text{GS},\max}, \forall t \in T, \forall i \in N^{\text{GS}} \quad (13)$$

c) *Gas Pipeline flow*: Gas flow in a pipeline from node i to node j can be represented as follows:

$$G_{ij,t}^{\text{pipe}} = \text{sgn}(\pi_{i,t}, \pi_{j,t}) K_{ij} \sqrt{|\pi_{i,t}^2 - \pi_{j,t}^2|}, \forall t \in T, \forall ij \in E^{\text{G}} \quad (14a)$$

$$\pi_i^{\min} \leq \pi_{i,t} \leq \pi_i^{\max}, \forall t \in T, \forall i \in N^{\text{G}} \quad (14b)$$

where K_{ij} is the pipeline constant that depends on the length and diameter of the pipeline, etc. The function $\text{sgn}(\pi_{i,t}, \pi_{j,t})$ represents the direction of gas flow in the pipeline ij at time t , which is defined as:

$$\text{sgn}(\pi_{i,t}, \pi_{j,t}) = \begin{cases} 1, \pi_{i,t} \geq \pi_{j,t} \\ -1, \pi_{i,t} < \pi_{j,t} \end{cases}, \forall t \in T, \forall i, \forall j \in N^{\text{G}} \quad (15)$$

where constraints (14b) describe the pressure limitation of gas node i .

d) *GSS*: Similar to the ESS constraints, the GSS constraints are described as follows:

$$\begin{aligned} u_{i,t}^{\text{GSS},d} + u_{i,t}^{\text{GSS},c} &\leq 1 \\ 0 &\leq G_{i,t}^{\text{GSS},d} \leq u_{i,t}^{\text{GSS},d} G_i^{\text{GSS},\max} \\ 0 &\leq G_{i,t}^{\text{GSS},c} \leq u_{i,t}^{\text{GSS},c} G_i^{\text{GSS},\max} \\ E_{i,t+1}^{\text{GSS}} &= E_{i,t}^{\text{GSS}} + \alpha_c^{\text{GSS}} G_{i,t}^{\text{GSS},c} - \alpha_d^{\text{GSS}} G_{i,t}^{\text{GSS},d} \\ E_i^{\text{GSS},\min} &\leq E_{i,t}^{\text{GSS}} \leq E_i^{\text{GSS},\max}, \forall t \in T, \forall i \in N^{\text{GSS}} \end{aligned} \quad (16)$$

3) Constraints of Heat Network

In this model, the heat energy required for the heat load is derived from EB, GB and WHB. In the heat network, the balance between the supply and consumption of heat energy needs to be satisfied:

$$\sum_{i \in N^{\text{GB}}} H_{i,t}^{\text{GB}} + \sum_{i \in N^{\text{WHB}}} H_{i,t}^{\text{WHB}} + \sum_{i \in N^{\text{EB}}} H_{i,t}^{\text{EB}} = H_t^{\text{L}}, t \in T \quad (17)$$

4) Constraints of Coupling Units

a) *P2G*: As the coupling link between the power network and the gas network, the model of P2G can be expressed as follows:

$$G_{i,t}^{\text{P2G}} = \eta_i^{\text{P2G}} P_{i,t}^{\text{P2G}} / H_{\text{GV}}, \forall t \in T, \forall i \in N^{\text{P2G}} \quad (18a)$$

$$P_{i,t}^{\text{P2G},\min} \leq P_{i,t}^{\text{P2G}} \leq P_{i,t}^{\text{P2G},\max}, \forall t \in T, \forall i \in N^{\text{P2G}} \quad (18b)$$

b) *GT*: As another coupling unit of the power and natural gas network, GT is the power source in the power network and the gas load in the gas network. The constraints of GT are expressed as follows:

$$P_{i,t}^{\text{GT}} = \eta_i^{\text{GT}} G_{i,t}^{\text{GT}} H_{\text{GV}} \quad (19a)$$

$$H_{i,t}^{\text{WHB}} = \lambda_i^{\text{GT}} P_{i,t}^{\text{GT}} \quad (19b)$$

$$u_{i,t}^{\text{GT}} P_i^{\text{GT},\min} \leq P_{i,t}^{\text{GT}} \leq u_{i,t}^{\text{GT}} P_i^{\text{GT},\max} \quad (19c)$$

$$u_{i,t}^{\text{GT}} G_i^{\text{GT},\min} \leq G_{i,t}^{\text{GT}} \leq u_{i,t}^{\text{GT}} G_i^{\text{GT},\max} \quad (19d)$$

$$-R_{\text{D}}^{\text{GT}} \leq P_{i,t}^{\text{GT}} - P_{i,t-1}^{\text{GT}} \leq R_{\text{U}}^{\text{GT}} \quad (19e)$$

$$y_{i,t}^{\text{GT}} - z_{i,t}^{\text{GT}} = u_{i,t}^{\text{GT}} - u_{i,t-1}^{\text{GT}} \quad (19f)$$

$$y_{i,t}^{\text{GT}} + z_{i,t}^{\text{GT}} \leq 1, \forall t \in T, \forall i \in N^{\text{GT}} \quad (19g)$$

In the above formulation, constraints (19b) describe the heating supply of the CHP system is proportional to its power

output. The ramp-up and ramp-down constraints are described in (19e). Constraints (19f) describe the relationship between the start-up/shut-down operations and the on/off statuses. Constraints (19 g) indicate GT cannot be started up and shut down simultaneously.

c) *EB*:

$$\begin{aligned} H_{i,t}^{\text{EB}} &= \eta_i^{\text{EB}} P_{i,t}^{\text{EB}} \\ H_{i,t}^{\text{EB}, \min} &\leq H_{i,t}^{\text{EB}} \leq H_{i,t}^{\text{EB}, \max}, \forall t \in T, \forall i \in N^{\text{EB}} \end{aligned} \quad (20)$$

d) *GB*:

$$\begin{aligned} H_{i,t}^{\text{GB}} &= \eta_i^{\text{GB}} G_{i,t}^{\text{GB}} H_{\text{GV}} \\ H_{i,t}^{\text{GB}, \min} &\leq H_{i,t}^{\text{GB}} \leq H_{i,t}^{\text{GB}, \max}, \forall t \in T, \forall i \in N^{\text{GB}} \end{aligned} \quad (21)$$

D. Second-stage Constraints

During the second-stage (the real-time operational stage), some flexible devices (with the symbol of Δ) should be adjusted from the day-ahead operational scheme formulated in the first stage because of the uncertainty of wind power output. Therefore, the redispatch problem based on the real-time scenarios needs to be considered at this stage. Notably, ESS and GSS maintain their outputs and the on/off status of GT remains unchanged. Constraints for the second-stage real-time optimal operation can be formulated as follows:

1) Constraints of Power Network

a) *BFM*:

$$\begin{aligned} \sum_{k \in \delta(j)} P_{jk,t,s} - \sum_{i \in \pi(j)} (P_{ij,t,s} - I_{ij,t,s}^2 r_{ij}) &= (P_{j,t}^{\text{ESS}, d} - P_{j,t}^{\text{ESS}, c}) \\ &+ (P_{j,t}^{\text{GT}} + \Delta P_{j,t}^{\text{GT}}) + P_{j,t}^{\text{W}} + (P_{j,t}^{\text{sub}} + \Delta P_{j,t}^{\text{sub}}) - (P_{j,t}^{\text{P2G}} \\ &+ \Delta P_{j,t}^{\text{P2G}}) - (P_{j,t}^{\text{EB}} + \Delta P_{j,t}^{\text{EB}}) - P_{j,t}^{\text{L}}, \forall t \in T, \forall j \in B \end{aligned} \quad (22a)$$

$$\begin{aligned} \sum_{k \in \delta(j)} Q_{jk,t,s} - \sum_{i \in \pi(j)} (Q_{ij,t,s} - I_{ij,t,s}^2 x_{ij}) &+ b_j V_{i,t,s}^2 = (Q_{j,t}^{\text{GT}} \\ &+ \Delta Q_{j,t}^{\text{GT}}) + (Q_{j,t}^{\text{sub}} + \Delta Q_{j,t}^{\text{sub}}) - Q_{j,t}^{\text{L}}, \forall t \in T, \forall j \in B \end{aligned} \quad (22b)$$

b) *Wind Power*:

$$0 \leq P_{i,t,s}^{\text{W}} \leq P_{i,t,s}^{\text{W},0}, \forall t \in T, \forall i \in N^{\text{W}} \quad (23)$$

c) *Substation Power*:

$$\begin{cases} P_t^{\text{sub}, \min} \leq P_t^{\text{sub}} + \Delta P_{t,s}^{\text{sub}} \leq P_t^{\text{sub}, \max} \\ Q_t^{\text{sub}, \min} \leq Q_t^{\text{sub}} + \Delta Q_{t,s}^{\text{sub}} \leq Q_t^{\text{sub}, \max} \end{cases}, \forall t \in T \quad (24)$$

2) Constraints of Gas Network

a) *Gas Balance*:

$$\begin{aligned} (G_{i,t}^{\text{GS}} + \Delta G_{i,t}^{\text{GS}}) + (G_{i,t}^{\text{GSS}, d} - G_{i,t}^{\text{GSS}, c}) + (G_{i,t}^{\text{P2G}} + \Delta G_{i,t}^{\text{P2G}}) \\ = G_{i,t}^{\text{L}} + \sum_{j \in i} (G_{ij,t}^{\text{pipe}} + \Delta G_{ij,t}^{\text{pipe}}) + (G_{i,t}^{\text{GT}} + \Delta G_{i,t}^{\text{GT}}) \\ + (G_{i,t}^{\text{GB}} + \Delta G_{i,t}^{\text{GB}}), \forall t \in T, \forall i, \forall j \in N^{\text{G}} \end{aligned} \quad (25)$$

b) *Gas Injection Capacity*:

$$G_{i,t}^{\text{GS}, \min} \leq G_{i,t}^{\text{GS}} + \Delta G_{i,t}^{\text{GS}} \leq G_{i,t}^{\text{GS}, \max}, \forall t \in T, \forall i \in N^{\text{GS}} \quad (26)$$

c) *Gas Pipeline flow*:

$$G_{ij,t}^{\text{pipe}} + \Delta G_{ij,t}^{\text{pipe}} = \text{sgn}(\pi_{i,t}, \pi_{i,t}) K_{ij} \quad (27a)$$

$$\begin{aligned} \sqrt{[(\pi_{i,t} + \Delta \pi_{i,t,s})^2 - (\pi_{j,t} - \Delta \pi_{i,t,s})^2]}, \forall t \in T, \forall ij \in E^{\text{G}} \\ \pi_i^{\min} \leq \pi_{i,t} + \Delta \pi_{i,t,s} \leq \pi_i^{\max}, \forall t \in T, \forall i \in N^{\text{G}} \end{aligned} \quad (27b)$$

3) Constraints of Heat Network

$$\begin{aligned} \sum_{i \in N^{\text{GB}}} (H_{i,t}^{\text{GB}} + \Delta H_{i,t}^{\text{GB}}) + \sum_{i \in N^{\text{WHB}}} (H_{i,t}^{\text{WHB}} + \Delta H_{i,t}^{\text{WHB}}) \\ + \sum_{i \in N^{\text{EB}}} (H_{i,t}^{\text{EB}} + \Delta H_{i,t}^{\text{EB}}) = H_t^{\text{L}}, \forall t \in T \end{aligned} \quad (28)$$

4) Constraints of Coupling Units

a) *P2G*:

$$P_i^{\text{P2G}, \min} \leq P_{i,t}^{\text{P2G}} + \Delta P_{i,t}^{\text{P2G}} \leq P_i^{\text{P2G}, \max}, \forall t \in T, \forall i \in N^{\text{P2G}} \quad (29)$$

b) *GT*:

$$\begin{aligned} u_{i,t}^{\text{GT}} P_i^{\text{GT}, \min} &\leq P_{i,t}^{\text{GT}} + \Delta P_{i,t}^{\text{GT}} \leq u_{i,t}^{\text{GT}} P_i^{\text{GT}, \max} \\ u_{i,t}^{\text{GT}} G_i^{\text{GT}, \min} &\leq G_{i,t}^{\text{GT}} + \Delta G_{i,t}^{\text{GT}} \leq u_{i,t}^{\text{GT}} G_i^{\text{GT}, \max} \\ -R_D^{\text{GT}} &\leq (P_{i,t}^{\text{GT}} + \Delta P_{i,t}^{\text{GT}}) - (P_{i,t-1}^{\text{GT}} + \Delta P_{i,t-1}^{\text{GT}}) \leq R_U^{\text{GT}} \\ \Delta H_{i,t}^{\text{GB}} &= \lambda_i^{\text{GT}} \Delta P_{i,t}^{\text{GT}}, \forall t \in T, \forall i \in N^{\text{GT}} \end{aligned} \quad (30)$$

c) *EB*:

$$H_{i,t}^{\text{EB}, \min} \leq H_{i,t}^{\text{EB}} + \Delta H_{i,t}^{\text{EB}} \leq H_{i,t}^{\text{EB}, \max}, \forall t \in T, \forall i \in N^{\text{EB}} \quad (31)$$

d) *GB*:

$$H_{i,t}^{\text{GB}, \min} \leq H_{i,t}^{\text{GB}} + \Delta H_{i,t}^{\text{GB}} \leq H_{i,t}^{\text{GB}, \max}, \forall t \in T, \forall i \in N^{\text{GB}} \quad (32)$$

IV. SOLUTION METHODOLOGY

In this section, the CCG algorithm is introduced to solve the DDRO model for UIES. Moreover, we propose effective linearization methods to deal with those nonlinear constraints and the absolute constraints which are difficult to be directly handled. Finally, SOCR is used for convex handling of the BFM model.

A. CCG Algorithm

The Benders' decomposition algorithm and the CCG algorithm are generally used to solve the multi-level optimization model. Compared with the Benders' decomposition algorithm, the CCG algorithm exhibits a better convergence performance [25], [26]. The details are outlined as follows.

The CCG algorithm divides the original distributionally robust model (4) into a master problem (MP) and a sub-problem (SP) to be solved iteratively. The iterations would stop until the difference of optimization results between MP and SP meets the predefined termination criterion.

The role of the MP is to find the optimal first-stage robust solution under some given finite worst-case probability distributions as well as provide a lower bound for (4):

$$\begin{aligned} \min_{\mathbf{x} \in \mathbf{X}, \mathbf{y}_0 \in \mathbf{Y}(\mathbf{x}, \xi_0), \mathbf{y}_s^{(m)} \in \mathbf{Y}(\mathbf{x}, \xi_s), W} \quad & \mathbf{a}^T \mathbf{x} + \mathbf{b}^T \mathbf{y}_0 + \mathbf{c}^T \xi_0 + W \end{aligned} \quad (33a)$$

$$W \geq \sum_{s=1}^S p_s^{(m)} (\mathbf{b}^T \mathbf{y}_s^{(m)} + \mathbf{c}^T \boldsymbol{\xi}_s), \forall m = 1, 2, \dots, n \quad (33b)$$

where $\mathbf{x}$ represents the first-stage robust decision variables; $\mathbf{y}_s$ and $\boldsymbol{\xi}_s$ represent the second-stage variables in scenario s and the wind power output for the clustered real-time scenario s , respectively; ξ_0 is the forecasted output for wind power; and m represents the number of iterations.

The subproblem is to generate the worst-case-scenario distribution based on the first-stage decision variables $\mathbf{x}^*$ given by the master problem and provide an upper bound for (4):

$$W(\mathbf{x}^*) = \max_{\{p_s\} \in \Omega} \sum_{s=1}^S p_s \min_{\mathbf{y}_s \in \mathbf{Y}(\mathbf{x}^*, \boldsymbol{\xi}_s)} (\mathbf{b}^T \mathbf{y}_s + \mathbf{c}^T \boldsymbol{\xi}_s) \quad (34)$$

Since each scenario is independent with each other, the above inner min function can be independently solved. If we use $f(\mathbf{x}^*, \boldsymbol{\xi}_s)$ to represent the optimal result for wind power scenario s , model (34) can be rewritten as the following linear programming model (35) to easily obtain the worst-case distribution $\{p_s^{(n)}\}$. This distribution can be passed to (33) to start the next iteration.

$$W(\mathbf{x}^*) = \max_{\{p_s\} \in \Omega} \sum_{s=1}^S f(\mathbf{x}^*, \boldsymbol{\xi}_s) p_s \quad (35)$$

B. Linearization Methods

1) Linearization of Confidence Set

The absolute constraints in (1) are converted to linear constraints for better convergence and shorter computational time. By taking norm-1 as an example, in order to facilitate the solution of the model, we introduce the 0–1 auxiliary variables z_s^+ and z_s^- to denote the offset status from the initial probability distribution p_s^0 .

$$z_s^+ + z_s^- \leq 1 \quad \forall s \quad (36a)$$

$$\begin{cases} 0 \leq p_s^+ \leq z_s^+ \theta_1 & \forall s \\ 0 \leq p_s^- \leq z_s^- \theta_1 & \forall s \\ p_s = p_s^0 + p_s^+ - p_s^- & \forall s \end{cases} \quad (36b)$$

where p_s^+ and p_s^- are the positive and negative deviations of the actual probability distribution p_s from the initial probability distribution p_s^0 , respectively. Based on this, the original constraint of norm-1 is converted to the following form:

$$\sum_{s=1}^S p_s^+ + p_s^- \leq \theta_1 \quad (37)$$

Similarly, the norm-inf constraint can be written as:

$$p_s^+ + p_s^- \leq \theta_\infty, \forall s \quad (38)$$

2) Linearization of Gas Pipeline flow

Constraints (14a) and (27a) are nonlinear, which also need to be linearized. For notational brevity, the time index t is omitted in this section. We introduce the 0–1 auxiliary variables u_{ij} and u_{ji} to denote the gas flows from bus i to bus j and bus j to bus i , respectively. Since the pipeline flow direction is unique, we impose the following supplementary constraints:

$$u_{ij} + u_{ji} \leq 1, \forall ij \in E^g \quad (39)$$

Then the gas flow direction $\text{sgn}(p_{i,t}, p_{j,t})$ in (15) can be rewritten as:

$$\text{sgn}(\pi_{i,t}, \pi_{j,t}) = u_{ij} - u_{ji}, \forall t \in T, \forall ij \in E^g \quad (40)$$

Square on both sides of (14a) and combine (40) with the squared equation. Then we can obtain (41):

$$(G_{ij}^{\text{pipe}})^2 = (u_{ij} - u_{ji}) K_{ij}^2 (\pi_i^2 - \pi_j^2), \forall ij \in E^g \quad (41)$$

When the direction of gas pipeline flow is from bus i to bus j , expression (41) is equivalent to:

$$(G_{ij}^{\text{pipe}})^2 = K_{ij}^2 (\pi_i^2 - \pi_j^2), \forall ij \in E^g \quad (42)$$

where π_i^2 can be regarded as a single variable in the optimization programming. So, the left-hand side of the equation is $(G_{ij}^{\text{pipe}})^2$ still nonlinear after the equivalent transformation because G_{ij}^{pipe} is already a variable. The piecewise-linearization is adopted to divide $(G_{ij}^{\text{pipe}})^2$ linearly into n^{pipe} segments as follows:

$$(G_{ij}^{\text{pipe}})^2 = \sum_{q=1}^{n^{\text{pipe}}} u_{ij,q} (G_{ij,q}^{\text{pipe}})^2, \forall ij \in E^g \quad (43)$$

When the pipeline flow direction is from bus j to bus i , the treatment method is similar, so we will not describe it here.

C. SOCR for BFM

It can be seen that the BFM constraints (7) requires SOCR for convex processing. We introduce the following equivalent transformation:

$$\begin{cases} \tilde{I}_{ij,t} = I_{ij,t}^2, & \forall t \in T, \forall ij \in E \\ \tilde{V}_{j,t} = V_{j,t}^2, & \forall t \in T, \forall j \in B \end{cases} \quad (44)$$

According to SOCR [27] and the above equivalent equation (44), the BFM model (7a)–(7d) can be represented as follows:

$$\begin{aligned} \sum_{k \in \delta(j)} P_{jk,t} - \sum_{i \in \pi(j)} (P_{ij,t} - \tilde{I}_{ij,t} r_{ij}) &= (P_{j,t}^{\text{ESS}, d} - P_{j,t}^{\text{ESS}, c}) \\ &+ P_{j,t}^{\text{GT}} + P_{j,t}^{\text{W}} + P_{j,t}^{\text{sub}} - P_{j,t}^{\text{P2G}} - P_{j,t}^{\text{EB}} - P_{j,t}^{\text{L}}, \forall t \in T, \forall j \in B \end{aligned} \quad (45a)$$

$$\begin{aligned} \sum_{k \in \delta(j)} Q_{jk,t} - \sum_{i \in \pi(j)} (Q_{ij,t} - \tilde{I}_{ij,t} x_{ij}) + b_j \tilde{V}_{i,t} \\ = Q_{j,t}^{\text{GT}} + Q_{j,t}^{\text{sub}} - Q_{j,t}^{\text{L}}, \forall t \in T, \forall j \in B \end{aligned} \quad (45b)$$

$$\begin{aligned} \tilde{V}_{j,t} = \tilde{V}_{i,t} - 2(P_{ij,t} r_{ij} + Q_{ij,t} x_{ij}) + \tilde{I}_{ij,t} (r_{ij}^2 + x_{ij}^2), \\ \forall t \in T, \forall ij \in E \end{aligned} \quad (45c)$$

$$\left\| \begin{matrix} 2P_{ij,t} \\ 2Q_{ij,t} \\ \tilde{I}_{ij,t} - \tilde{V}_{i,t} \end{matrix} \right\|_2 \leq \tilde{I}_{ij,t} + \tilde{V}_{i,t}, \forall t \in T, \forall ij \in E \quad (45d)$$

Finally, the relaxed conic power flow constraints (45a)–(45d) are included to replace (7) in the UIES. The relaxation exactness of SOCR for the optimal power flow model under radial network is verified in [27]. The accuracy of the BFM model under a circular network is also studied [24], [28]. If the relaxation is not tight, we can use the cutting plane method to obtain the exact solution of the relaxed optimal power flow model [29]. The cutting plane can reduce the relaxation

distance by adding new cuts until the gap finally meets the iterative convergence requirement.

After performing the above procedure, a mixed-integer second-order cone programming (MISOCP) reformulation can be obtained and easily solved using commercial solvers, such as CPLEX.

V. CASE STUDIES

In this section, we apply a modified IEEE 33-bus system with a 6-node natural gas system to test the effectiveness of the proposed model. All the experiments are implemented in MATLAB R2016a with CPLEX 12.6 on a 3.0 GHz computer with 4 GB memory.

A. Data Configuration

The penalty price of wind power curtailed is \$1/(kW·h). The electricity price is obtained from [24]. We assume that 1 kilo-cubic feet of natural gas can generate 1 MBtu of heat energy [30], and the price of natural gas is \$5/MBtu (i.e., \$5/kcf). The topology is depicted in Fig. 2. The power system is derived from the IEEE 33-bus system. The modified 6-node natural gas system from [30] with 5 pipelines, 2 GSs, 2 GSSs and 2 GBs. The forecasted values of the WTG output and electricity load are depicted in Fig. 3, and the data of the heat load and gas load are shown in Fig. 4. The detailed parameters for GT, P2G, EB, GB, ESS and GSS are listed in Tables AI–AVII in Appendix A.

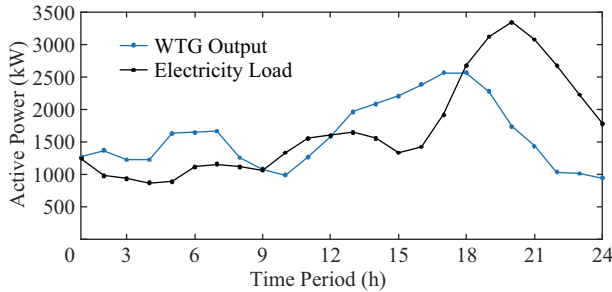


Fig. 3. Daily curves of WTG output and electricity load.

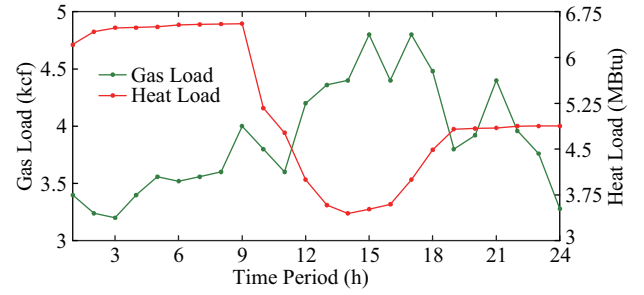


Fig. 4. Daily curves of gas load and heat load.

To generate the historical data samples, we assume that the uncertain wind power output follows a multivariate normal distribution with the mean value as shown in Fig. 3 and the variance as 1/3 of the mean. We generate a large number of samples according to this distribution and treat them as the historical data in this case study. The K-means clustering algorithm is applied to obtain several independent scenarios. We set the number of independent scenarios to be 10.

B. Optimal Operational Results Analysis

Let the number of historical data samples be 1,000 and the confidence levels α_1 and α_∞ be 0.5 and 0.9, respectively. The output of power equipment (positive value indicates power supply and negative value indicates power consumption) and heat generating equipment in UIES are shown in Figs. 5 and 6. The charge/discharge curves of energy storage systems are shown in Fig. 7. Figure 8 depicts the substation power and the GS gas supply (injection) curves.

From Figs. 5–8, we can observe that in the first half of the day, the WTG output is relatively higher ($t = 1-9$ h) or slightly lower ($t = 9-12$ h) than the load. The power provided by GT and WTG can almost satisfy the load demand and little power is purchased from the substation. At this stage, the gas load is low and the heat load is somehow in the peak periods. Therefore, surplus electricity is preferentially used by EB to generate heat and the P2G output is low. Because GT and WHB constitute a CHP system, the output of WHB is proportional to GT. Since the power load is higher than the wind power in time periods $t = 13$ h–17 h, P2G converts more

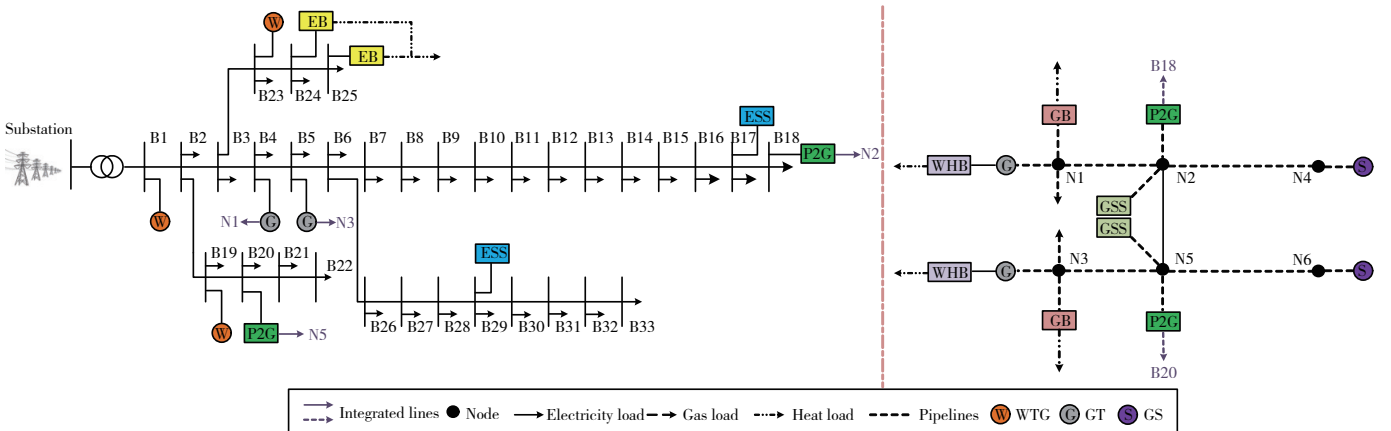


Fig. 2. The topology of UIES.

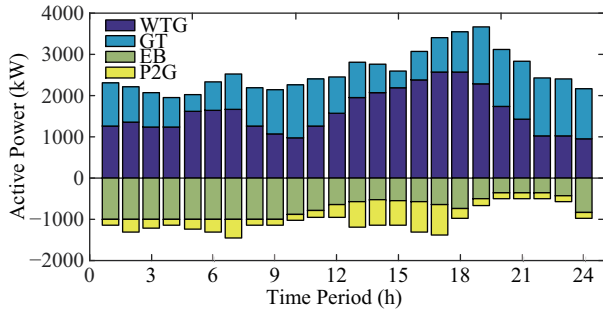


Fig. 5. The output of power equipment.

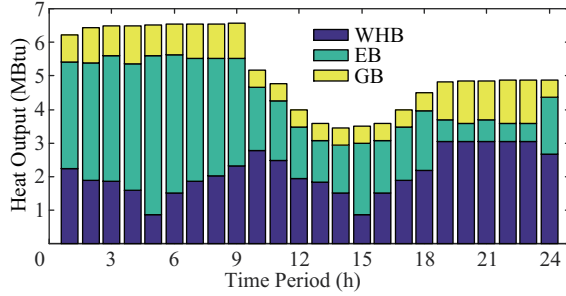


Fig. 6. The output of heat generating equipment.

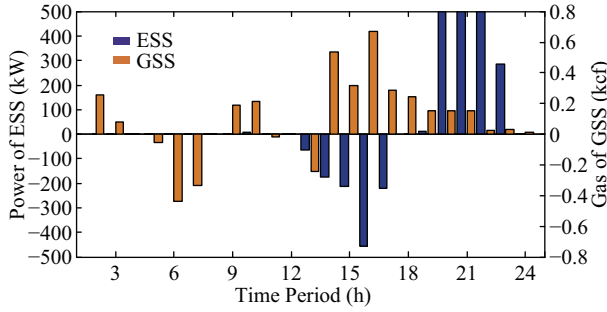


Fig. 7. The charge/discharge histogram of energy storage systems.

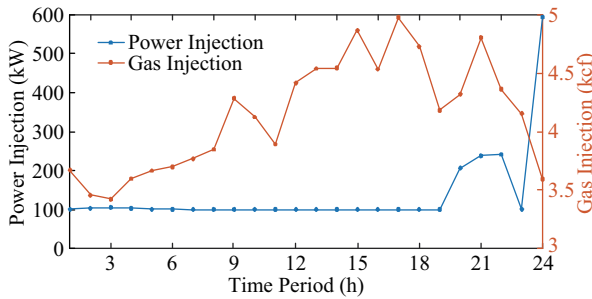


Fig. 8. Power and gas injection curves of UIES.

surplus power to natural gas to supply the gas load. At the same time, ESS can also store a certain amount of electricity energy. However, since the gas load is at its peak, GSS would discharge the energy to meet the load demand instead of storing natural gas during this period, which indicates that energy storage systems are able to shift some loads to valley periods. In time period $t = 18$ h–24 h, the WTG output is relatively lower compared to peak load, the demand for load cannot be balanced within the UIES and thus a higher

electricity purchasing from the substation occurs even if the electricity price is relatively high at this time. Because of the peak load, surplus electricity is scarce, P2G output is reduced and ESS begins to discharge the energy stored in the load valley. In addition, the GT power is in a high amount state to supply the electricity load and thus the heat output of WHB is also high because it is proportional to the GT power. In this stage, the EB output significantly declined because of the high heating supplied by WHB.

In order to verify the effects of the integrated energy system on wind power utilization, we devise another case (case 2), where GSS, ESS, EB and P2G are not considered. This case weakens the coupling between electricity, gas and heat networks. Wind power output of the two cases are shown in Fig. 9.

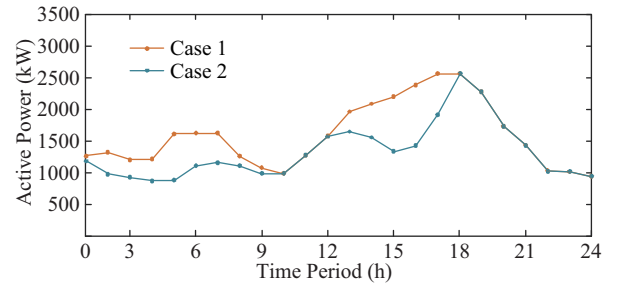


Fig. 9. Wind power output curve.

As shown in Fig. 9, during the entire time-periods, comparing the two cases, wind power curtailed of UIES in case 1 is only 0.9 kW·h, the actual wind power output is close to the predicted value. In case 2, since there is no coupling device, wind power is only used to supply the electrical load and cannot be converted into other forms of energy. Therefore, wind power curtailed in case 2 is increased to 6319.25 kW·h, which indicates that the efficiency of wind energy utilization is greatly improved in UIES, where the wind power can be utilized and transformed into other forms of energy via storage systems and those coupling units to solve the problem of wind curtailment.

C. Analysis of Effect of Historical Data on Results

Equation (3) shows that the conservativeness of the DDRO model is affected by the number of historical data samples. Therefore, the effect of the historical data on the results is tested in this section. Three cases (norm-1, norm-inf and co-constraints) are devised. The numbers of historical data vary from 100 to 5,000. Confidence levels of α_1 and α_∞ are set as 0.5 and 0.99, respectively.

TABLE I
EFFECTS OF HISTORICAL DATA AND NORMS

Number of historical data	Cost (\$)		
	Norm-1	Norm-inf	Co-constraints
100	1632.15	1630.32	1621.14
500	1599.69	1597.83	1588.01
1000	1598.85	1595.11	1586.26
2000	1593.80	1593.09	1583.78
5000	1591.89	1589.35	1579.56

As can be seen from Table I, the operational costs of UIES decreases with the increase in the number of historical data under all those three types of norms. This is because, as can be seen from (3), when the number of historical data increases, the fluctuation limits θ_1 and θ_∞ correspondingly decrease. As a result, the confidence set Ω of wind power probability distribution becomes smaller, which leads to the wind power output in real-time operations being closer to the predicted value. Therefore, the adjustable cost is reduced (i.e., the conservativeness of the model decreases). Consequently, the conservativeness of the DDRO model can be adjusted according to the given number of historical data. From the table we can also observe that in the case of the same number of historical data, compared to using a single norm, the operational cost of the system is lower when norm-1 and norm-inf are simultaneously used as constraints. It is shown that a smaller fluctuation value can be obtained by simultaneously using two norms. Thus, the model proposed in this paper is more economical.

D. Analysis of Confidence Level

In this section, we analyze the influence of confidence levels α_1 and α_∞ on the optimization results. In order to guarantee the robustness of the system, the number of historical data is set to 1,000. The results are shown in Table II.

TABLE II
EFFECTS OF CONFIDENCE LEVEL

α_1	Cost (\$)		
	$\alpha_\infty = 0.5$	$\alpha_\infty = 0.8$	$\alpha_\infty = 0.99$
0.5	1583.24	1584.58	1586.26
0.8	1586.27	1587.55	1588.72
0.99	1588.72	1589.62	1589.81

Table II shows that, the operational cost of UIES increases, with the increase of the confidence level. This is because when the confidence level increases, the confidence set Ω of the wind power probability distribution becomes larger, which leads to the increase of uncertainty in the wind power output. Thus, the system needs to adjust more units to cope with the uncertainty, the adjustable cost increases (i.e., the conservativeness of the model increases). Similarly, the conservativeness of the DDRO model can also be adjusted according to the given confidence level. UIES operators can choose different conservativeness based on the desired confidence levels.

E. Comparing DDRO with SO and RO

In order to investigate the effectiveness of the proposed DDRO model in this paper, we compare the DDRO with SO and RO in this section. We tested the system performance of the three approaches in two steps: 1) the decision variables of day-ahead scheduling are solved for by three algorithms respectively, and then the cost of the first stage is obtained; and 2) 5,000 wind power probability distribution scenarios are randomly generated for comparative testing and analysis of the second-stage adjustment costs. For the DDRO model, we set α_1 as 0.5, α_∞ as 0.99, and the number of historical data is also 1,000. Results of the costs for the three approaches are given in Table III.

TABLE III
COMPARISON WITH SO AND RO

Types	First stage cost (\$)	First stage Time (s)	Adjustment cost (\$)		Time (s)
			Mean value	Maximum value	
DDRO	1585.77	361	4.30	7.32	2487
SO	1578.94	255	7.14	9.73	1846
RO	1715.71	387	12.07	13.67	2511

As can be seen from Table III, the cost of the DDRO model in the first stage is between that of SO and RO. Compared with SO, the model proposed in this paper is more robust. UIES handles the uncertainties of wind power by adjusting the output of the coupling devices (GT, P2G, EB, GB, etc.) and natural gas supply. Compared with RO, the model proposed in this paper is more economical. This is because the RO approach generates the optimization scheme under the worst-case wind power scenario. Moreover, we can observe that the mean value and maximum value in the second stage in the model proposed in this paper are lower than that in the other two methods. This indicates DDRO's average and worst-case probability distribution performance are both better. Finally, as can be seen from the computation time, the computation efficiency of the DDRO method is also moderate. In summary, the DDRO approach combines SO and RO to achieve an equilibrium in economy and robustness.

F. Analysis of the Accuracy of the SOCR

In order to analyze the accuracy of the SOCR, we define the relative gap of the SOC relaxation for branch ij at time period t as follows:

$$\Delta_{ij,t}^{\text{diff}} = \left| \frac{P_{ij,t}^2 + Q_{ij,t}^2 - \tilde{I}_{ij,t} \tilde{V}_{j,t}}{P_{ij,t}^2 + Q_{ij,t}^2} \right| \quad (46)$$

Figure 10 depicts the relaxation gap of SOCR in different periods. As depicted in Fig. 10, We can see that the maximum gap stays within 10^{-3} . This indicates that the proposed SOCR method is effective.

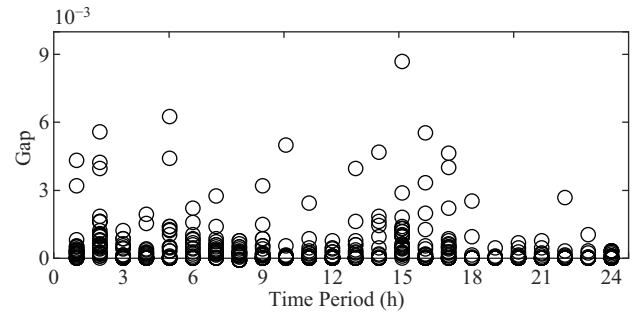


Fig. 10. Gap scatter of SOCR.

VI. CONCLUSION

This work proposes a two-stage DDRO model to handle the operations for UIES with uncertain wind power output considered. The confidence set for the probability distributions of the uncertain wind power is constructed by norm-1 and norm-inf co-constraints. The following conclusions can be drawn from the case studies.

1) The volatility of wind power output can be effectively mitigated in UIES where wind energy can be utilized and transformed into other forms of energy via storage systems and coupling devices to solve the problem of wind curtailment.

2) The number of historical data and confidence levels have a significant impact on the DDRO results. The conservativeness of the DDRO model can be adjusted according to the given number of historical data and confidence levels.

3) Norm-1 and norm-inf co-constraints exhibit a better economic performance compared to single norm constraints.

4) Compared to SO and RO models, the DDRO model shows a better performance. The DDRO approach combines SO and RO to achieve an equilibrium in economy and robustness.

In addition, the proposed DDRO method can be generally applied to model the uncertainty of other resources (e.g. photovoltaic, etc.). Some other stricter and pure mathematical linearization methods need to be further studied to deal with the polyhedral confidence set. In addition, the demand response and reserve would be included in the scheduling model of the integrated energy system to improve the system reliability in a future work.

APPENDIX A

TABLE AI
PARAMETERS OF GTs

Types	Lower (kW)	Upper (kW)	Conversion efficiency	Ramp (kW/h)
GT1	300	800	0.4	300
GT2	100	600	0.45	100

TABLE AII
COST DATA OF GTs

Types	a (MBtu/kW ² h)	b (MBtu/(kW·h))	c (MBtu)	Start-up cost (\$)	Shut-down cost (\$)
GT1	0.00016	0.67	58.81	100	90
GT2	0.00018	0.85	31.67	80	60

TABLE AIII
PARAMETERS OF P2Gs

Types	Lower (kW)	Upper (kW)	Conversion efficiency
P2G1	0	500	0.8
P2G2	0	600	0.8

TABLE AIV
PARAMETERS OF EBS

Types	Lower (kW)	Upper (kW)	Conversion efficiency
EB1	0	600	0.98
EB2	0	400	0.95

TABLE AV
PARAMETERS OF GBs

Types	Lower (MBtu)	Upper (MBtu)	Conversion efficiency
GB1	0	1.4	0.9
GB2	0	2	0.9

TABLE AVI
PARAMETERS OF ESS

Types	$E_i^{\text{ESS}, \min}$ (kW·h)	$E_i^{\text{ESS}, \max}$ (kW·h)	$P_i^{\text{ESS}, \max}$ (kW)	α_c^{ESS}	α_d^{ESS}
ESS1	100	1000	250	0.9	1.1
ESS2	50	1000	200	0.9	1.1

TABLE AVII
PARAMETERS OF GSS

Types	$E_i^{\text{GSS}, \min}$ (Kcf)	$E_i^{\text{GSS}, \max}$ (Kcf)	$G_i^{\text{GSS}, \max}$ (Kcf/h)	α_c^{GSS}	α_d^{GSS}
GSS1	0.1	4	1	0.9	1.11
GSS2	0.05	3	1	0.9	1.11

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